Control of a Flexible Joint

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Cours Automatique/ Control systems

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Module 1: Analysis of Feedback Control Systems

1.2 Give the transfer function of the flexible joint G(s) in zpk format :

$$G(s) = \frac{3654.6}{s(s+43.33)(s^2+6.82s+234.6)}$$

1.3.1 Write the transfer function S between r and e as a function of Dc and G and give its numerical values in zpk format.

$$S = \frac{E(s)}{R(s)} = \frac{1}{1+KG} = \frac{s(s+43.33)(s^2+6.82s+234.6)}{(s+42.94)(s+3.273)(s^2+3.928s+208)}$$

1.3.2 Write the transfer function U between r and u as a function of Dc and G and give its numerical values in zpk format.

$$U = \frac{U(s)}{R(s)} = \frac{K}{1+KG} = \frac{8 \cdot s(s+43.33)(s^2+6.82s+234.6)}{(s+42.94)(s+3.273)(s^2+3.928s+208)}$$

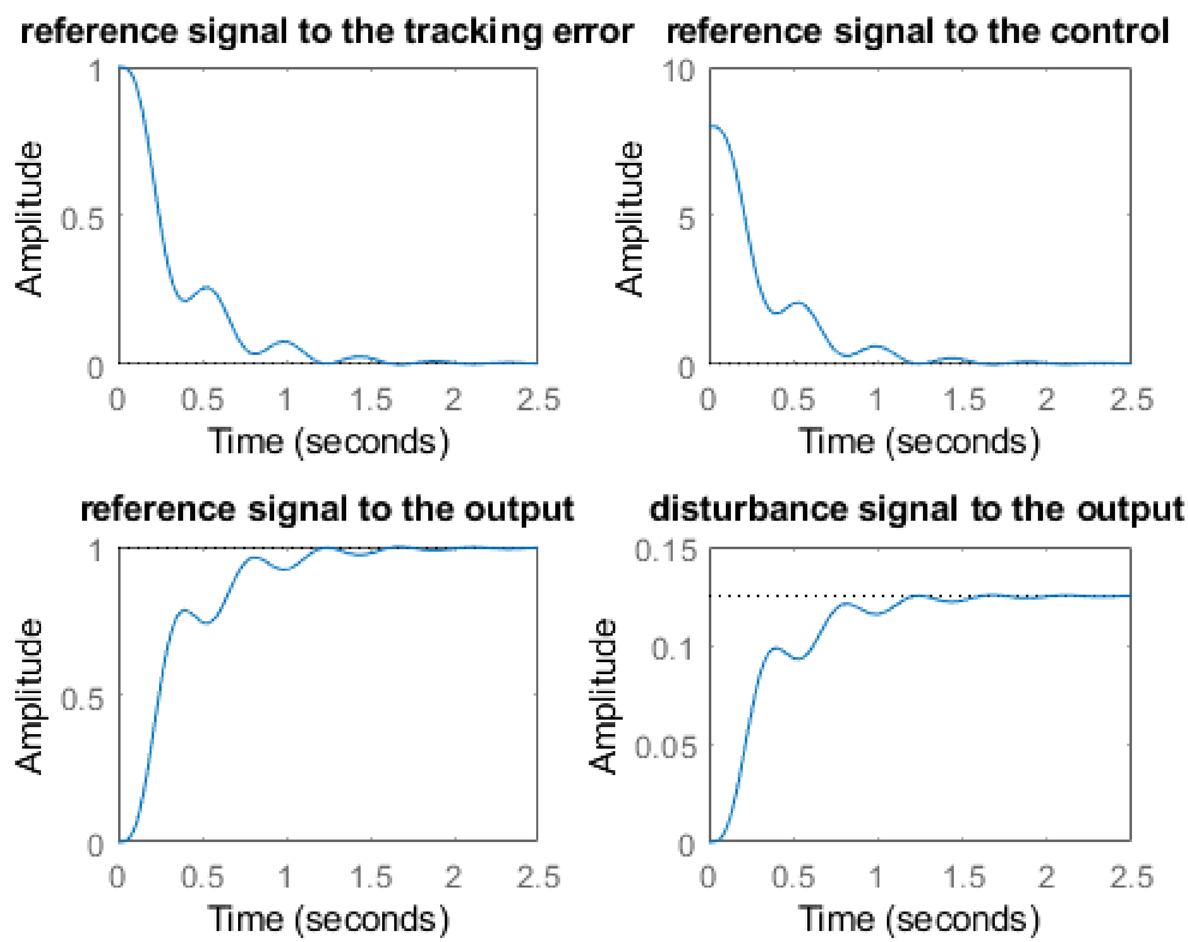
1.3.3 Write the transfer function T between r and y as a function of Dc and G and give its numerical values in zpk format.

$$T = \frac{Y(s)}{R(s)} = \frac{KG}{1+KG} = \frac{29'237}{(s+42.94)(s+3.273)(s^2+3.928s+208)}$$

1.3.4 Write the transfer function V between w and y as a function of Dc and G and give its numerical values in zpk format.

$$V = \frac{Y(s)}{W(s)} = \frac{G}{1+KG} = \frac{3'654.6}{(s+42.94)(s+3.273)(s^2+3.928s+208)}$$

1.3.a **Plot the step responses of the closed-loop system from reference signal to the output, from reference signal to the control signal (plant input), from reference signal to the tracking error signal (the input of the controller) and from disturbance signal to the output. All plots in one figure with appropriate scale (Use a 2 by 2 subplot).**

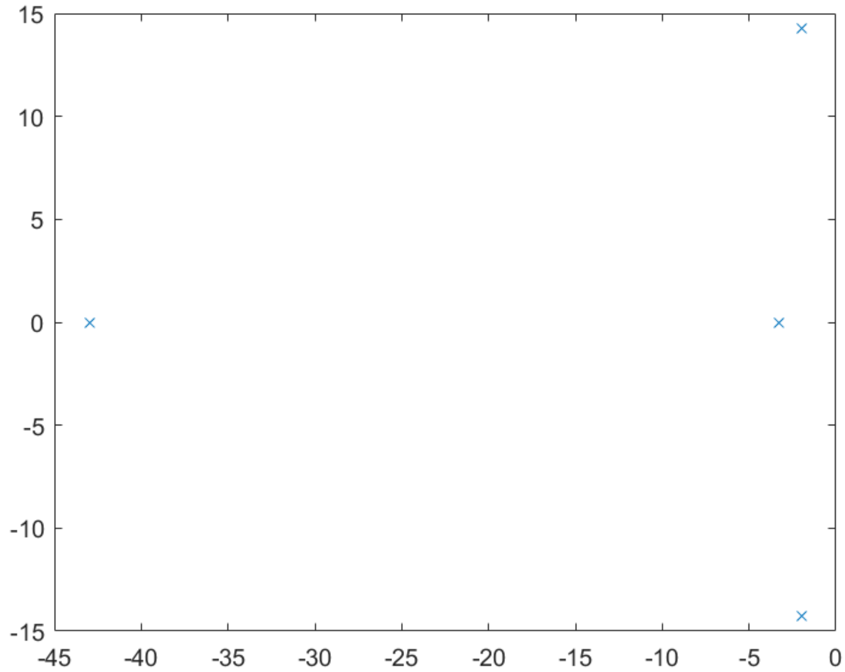


1.3.b **Give the closed-loop poles.**

- -42.9445
- $-1.9641 + 14.2873i$
- $-1.9641 - 14.2873i$
- -3.2734

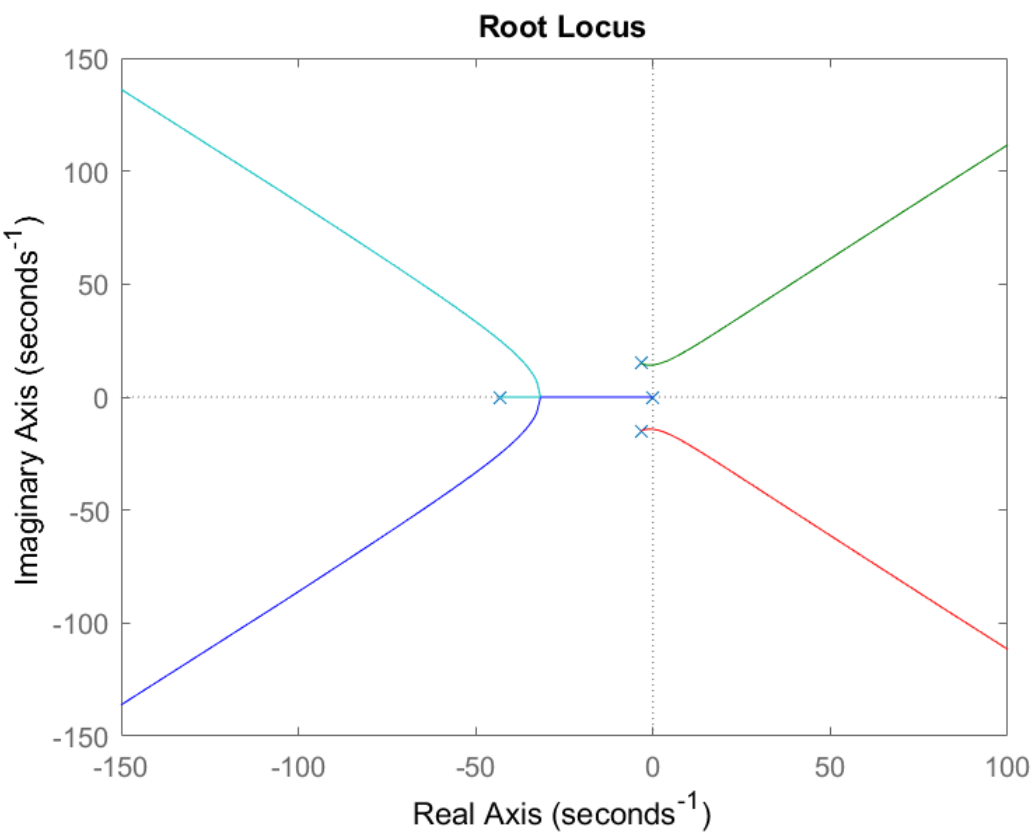
1.3.c **Is the closed-loop system stable? Why?**

The closed-loop system is stable because all the poles are located in the LHP.



1.4 Compute the ultimate gain using the plot of rlocus command of Matlab. Validate your result using the Routh stability criterion.

Using *rlocus(G,17:0.01:19)*, we can define precisely the value of K_u.



$$K_u \approx 18.2$$

We can apply the Routh criterion on $A(s) = s^4 + 50.15 \cdot s^3 + 530.1 \cdot s^2 + 1.016 \cdot 10^4 \cdot s + 3655 \cdot K$.

Therfore $\quad a_1 = 50.15 \quad a_2 = 530.1 \quad a_3 = 1.016 \cdot 10^4 \quad a_4 = 3655 \cdot K$

$$\begin{array}{c|ccc} s^4 & 1 & a_2 & a_4 \\ s^3 & a_1 & a_3 & a_5 \\ s^2 & b_1 & b_2 & b_3 \\ s & c_1 & c_2 & \\ 1 & d_1 & & \\ 0 & 0 & & \end{array}$$

$$b_1 = -\frac{det\begin{vmatrix} 1 & a_2 \\ a_1 & a_3 \end{vmatrix}}{a_1} = 327.51 \qquad b_2 = -\frac{det\begin{vmatrix} 1 & a_4 \\ a_1 & a_5 \end{vmatrix}}{a_1} = 3655 \cdot K$$

$$c_1 = -\frac{det\begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix}}{b_1} = -559.7 \cdot K + 1.016 \cdot 10^4 \qquad c_2 = -\frac{det\begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix}}{b_1} = 0$$

$$d_1 = -\frac{det\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{c_1} = 3655 \cdot K \qquad e_1 = -\frac{det\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}}{d_1} = 0$$

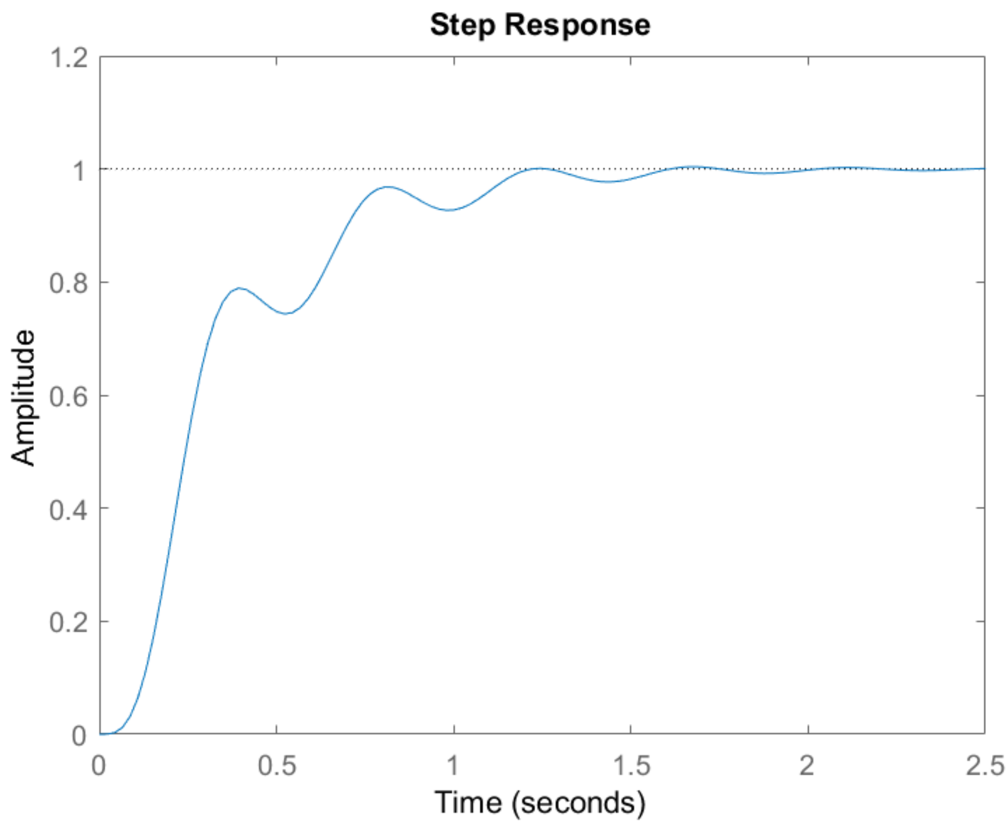
The Routh Criterion says the following:

- $1 > 0$
- $a_1 = 50.15 > 0$
- $b_1 = 327.51 > 0$
- $c_1 = -559.7 \cdot K + 1.016 \cdot 10^4 > 0$
- $d_1 = 3655 \cdot K > 0$

Therefore, we can calculate the value the ultimate gain: $K_u \approx 18.1536$.

The value correspond to the value calculated with the rlocus function in Matlab.

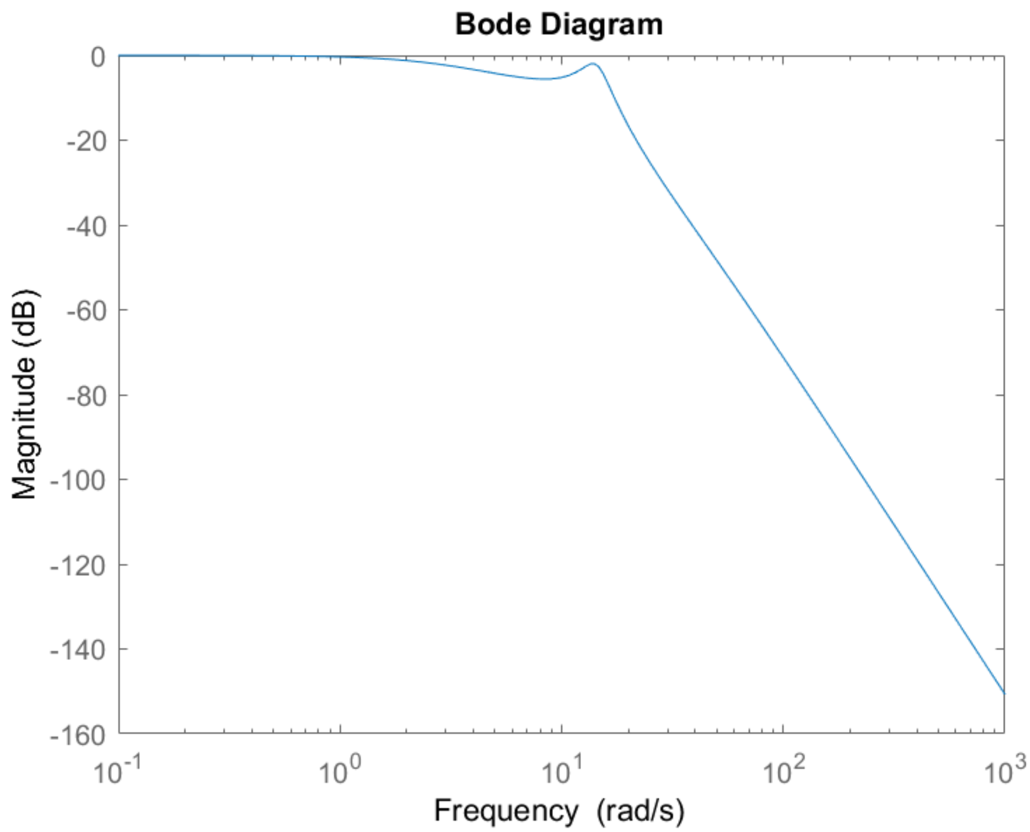
1.5 Plot the step response of the closed-loop system (between r and y). Print the rise-time, settling time and the overshoot (use stepinfo). Compute the closed-loop bandwidth (use bandwidth). Plot the magnitude Bode diagram of the closed-loop transfer function (use bodemag) and check the correctness of the bandwidth.



Using *stepinfo* and *bandwidth* gives us the following values:

- $Risetime = 0.5742s$
- $Settlingtime = 1.4827s$
- $Overshoot = 0.3937s$
- $Bandwidth = 3.6610$

The bode diagram of the system is:



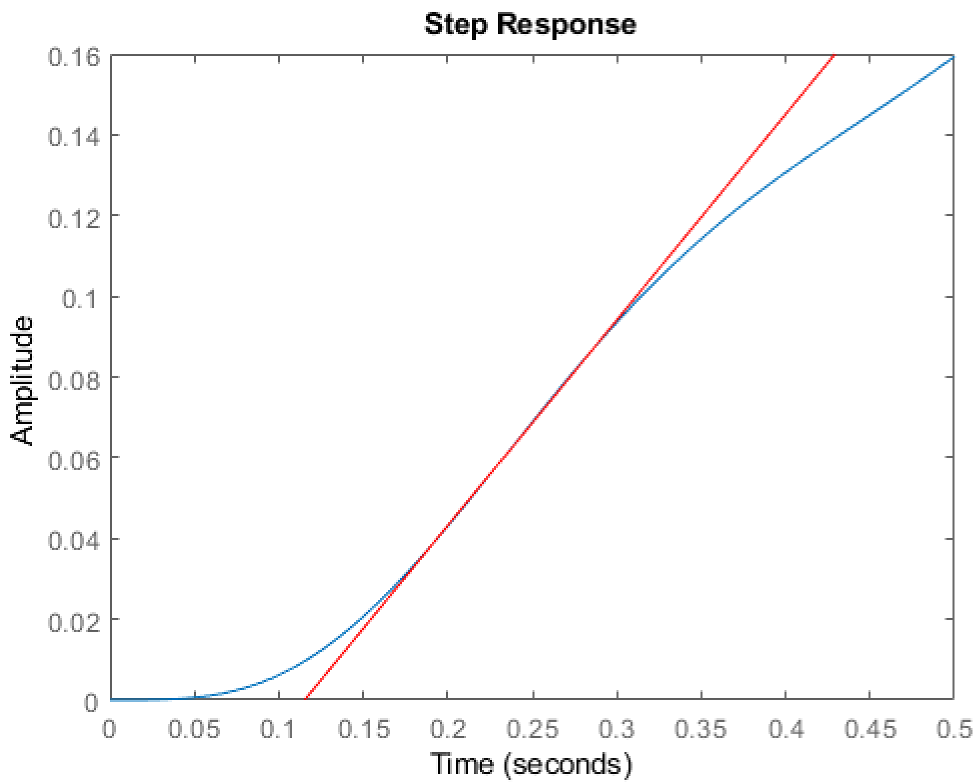
The bandwidth is defined for the value at which the system reaches the magnitude of $-3dB$. As a result, we can confirm the previous value by zooming on the graph.

Module 2: PID Controller Design

2.1 ZN First method

2.1.1 Plot the step response of G from 0 to 0.5 s together with the asymptote (or the tangent with the largest slope) for the first method of ZN.

The step response of $G(s)$ is:



2.1.2 Give the value of L and R.

The equation of the tangent is $A(t) = 1.9620 \cdot t + 0.1153$. Thus:

- $R = 1.9620$
- $L = 0.1153$

2.1.3 Give the parameters of the PID controller

The parameters are:

- $K_p = 5.3046$
- $T_i = 0.2306$
- $T_d = 0.0577$

2.1.4 Is the closed-loop system stable with this PID controller?

The new transfer function is:

$$T_{PID} = \frac{0.005684 \cdot s^2 + 0.1232 \cdot s + 0.4272}{5.052 \cdot 10^(- 6) \cdot s^5 + 0.0002548 \cdot s^4 + 0.002694 \cdot s^3 + 0.05734 \cdot s^2 + 0.1232 \cdot s + 0.4272}$$

The system is stable because all of its poles belong to the LHP:

- $-43.6554 + 0.0000i$
- $-2.2116 + 14.9773i$

- $-2.2116 - 14.9773i$
- $-1.0338 + 2.7079i$
- $-1.0338 - 2.7079i$

2.2 ZN Second method

2.2.1 Give the value of the ultimate gain and the ultimate period.

As calculated in question 1.4:

- $K_u = 18.1536$
- $P_u = 0.348s$,measured with the plot of the step response of the system with K_u

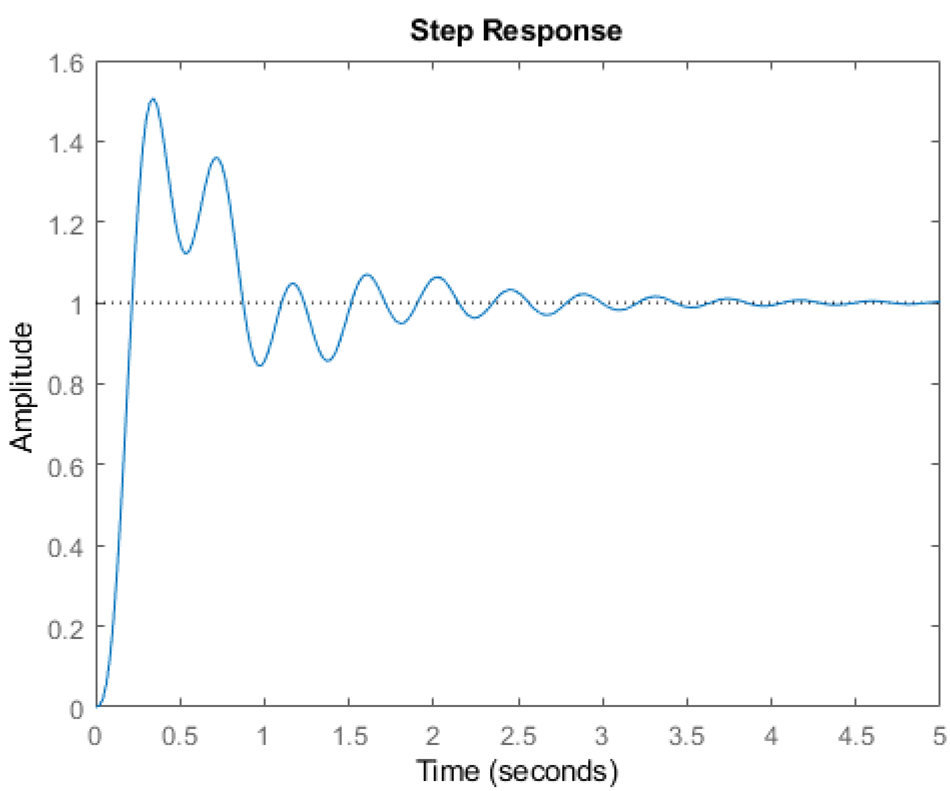
2.2.2 Give the parameters of the PID controller

The parameters are:

- $K_p = 10.892$
- $T_i = 0.174$
- $T_d = 0.044$

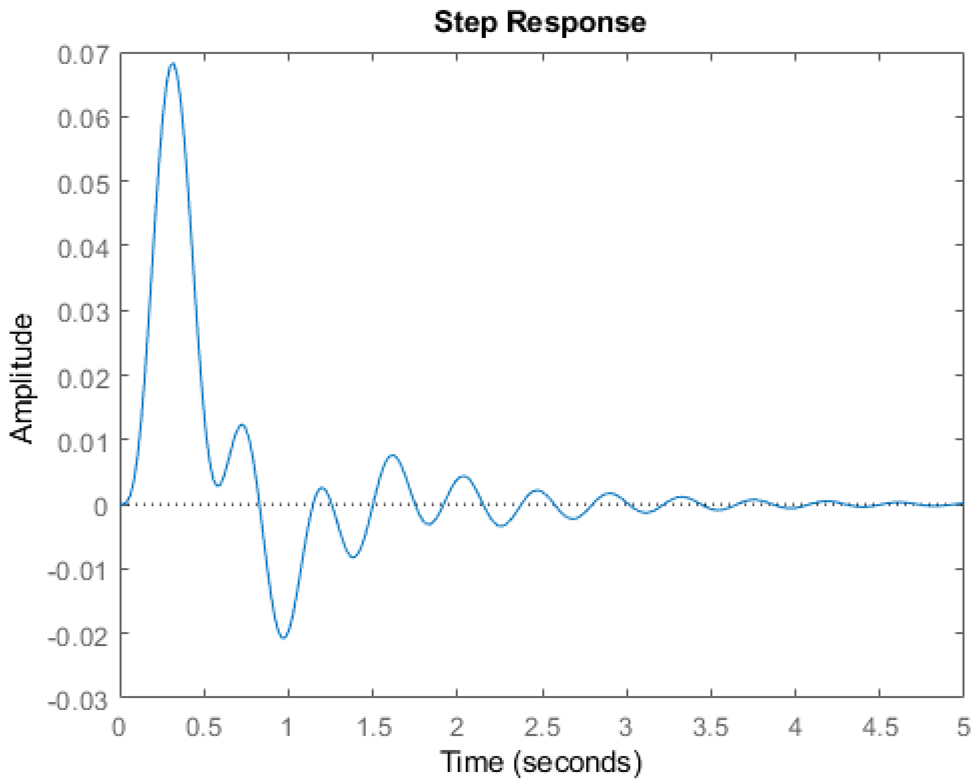
2.2.3 Plot the step response of the closed-loop system from the reference signal to the output

The step response of $\frac{Y(s)}{R(s)}$ using these parameters is:



2.2.4 Plot the step response of the closed-loop system from the disturbance signal to the output.

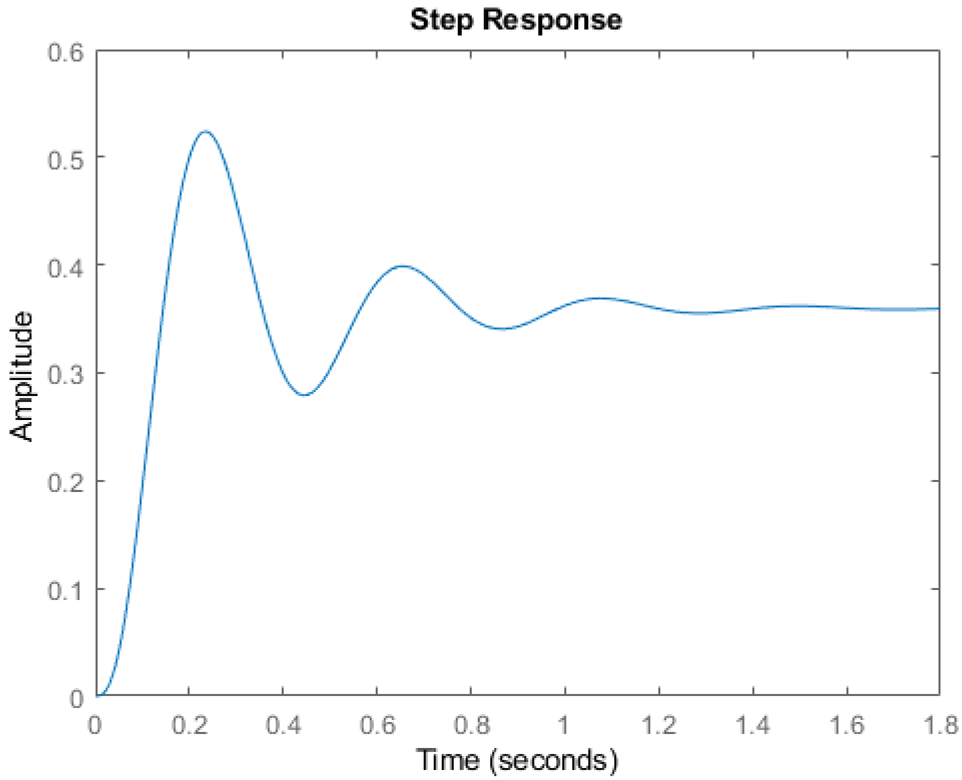
The step response of $\frac{Y(s)}{W(s)}$ using these parameters is:



2.3 Cascade Controller

2.3.1 Plot the step response of G1 and give the step information. From the step response, estimate the steady-state gain, the damping factor and the natural frequency of an approximate second-order model

$G_1(s)$ is defined as $G(s) \cdot s$ and its step response is the following:



These are the specifications given by *stepinfo*:

- $RiseTime = 0.0873$
- $SettlingTime = 1.1234$
- $SettlingMin = 0.2793$
- $SettlingMax = 0.5239$
- $Overshoot = 45.7139$
- $Undershoot = 0$
- $Peak = 0.5239$
- $PeakTime = 0.2338$

Graphically, we can estimate the steady state gain : $K_{ss} = 0.359$.

Using the $Peak$ and Kss values, we can calculate:

$$M_p = \frac{Peak - K_{ss}}{K_{ss}} = 0.4593$$

and deduce the damping factor:

$$\zeta = \sqrt{\frac{ln^2(M_p)}{\pi^2 + ln^2(M_p)}} = 0.2404$$

Knowing the *PeakTime*, we can compute the *Natural frequency*:

$$\omega_n = \frac{\pi}{PeakTime \cdot \sqrt{1-\zeta^2}} = 13.843$$

2.3.2 Give the reference model for the inner loop. Give the PID controller parameters of the inner loop.

Knowing that τ_m correspond to the inverse of the desired bandwidth ($40 rad/s$), we can set the reference model as

$$M(s) = \frac{1}{1+\tau_m \cdot s} = \frac{1}{1+0.025 \cdot s}$$

Knowing that $\alpha = 1$ is the amplitude of the applied step response, we can calculate : $\gamma = \frac{K_{ss}}{\alpha} = K_{ss}$
The parameters of the PID controller with this model can be calculated as follow:

- $k_p = \frac{2\zeta}{\gamma\omega_n\tau_m} = 3.8696$
- $k_i = \frac{1}{\gamma\tau_m} = 111.4206$
- $k_d = \frac{1}{\gamma\omega_n^2\tau_m} = 0.5814$

2.3.3 Give the reference model for the outer loop. Give the proportional controller for the outer loop.

Using the same procedure with a desired bandwidth of $4 rad/s$, we obtain:

- $M(s) = \frac{1}{1+\tau_m \cdot s} = \frac{1}{1+0.25 \cdot s}$
- $k_p = D_c(s) = \frac{1}{\gamma\tau_m} = 4$

2.3.4 Give the transfer function between R and Y in terms of Dc, Dc' and G. Give also the transfer function with its numerical values in zpk format.

$$T_{cascade} = \frac{Y(s)}{R(s)} = \frac{D_c D_c' G_1(s)}{s + D_c' G_1(s) \cdot s + D_c D_c' G_1(s)}$$

We know that $D_c'(S)$ can be computed as follow: $D_c' = k_p + k_d \cdot s + \frac{k_i}{s} = 3.8696 + 0.5814 \cdot s + \frac{111.4206}{s}$. As a result, we can calculate the numerical value and give its zpk format:

$$= \frac{8499.2 \cdot s^5 (s+43.33)^2 (s^2 + 6.656 \cdot s + 191.6)(s^2 + 6.82 \cdot s + 234.6)^2}{s^5 (s+43.33)^2 (s+4.446)(s^2 + 7.492 \cdot s + 185)(s^2 + 6.82 \cdot s + 234.6)^2 (s^2 + 38.21 \cdot s + 1980)}$$

2.3.5 Give the transfer function between W and Y in terms of Dc, Dc' and G. Give also the transfer function with its numerical values in zpk format.

We have the follwing calculated values:

- $D_c' = k_p + k_d \cdot s + \frac{k_i}{s} = 3.8696 + 0.5814 \cdot s + \frac{111.4206}{s}$
- $D_c = 4$

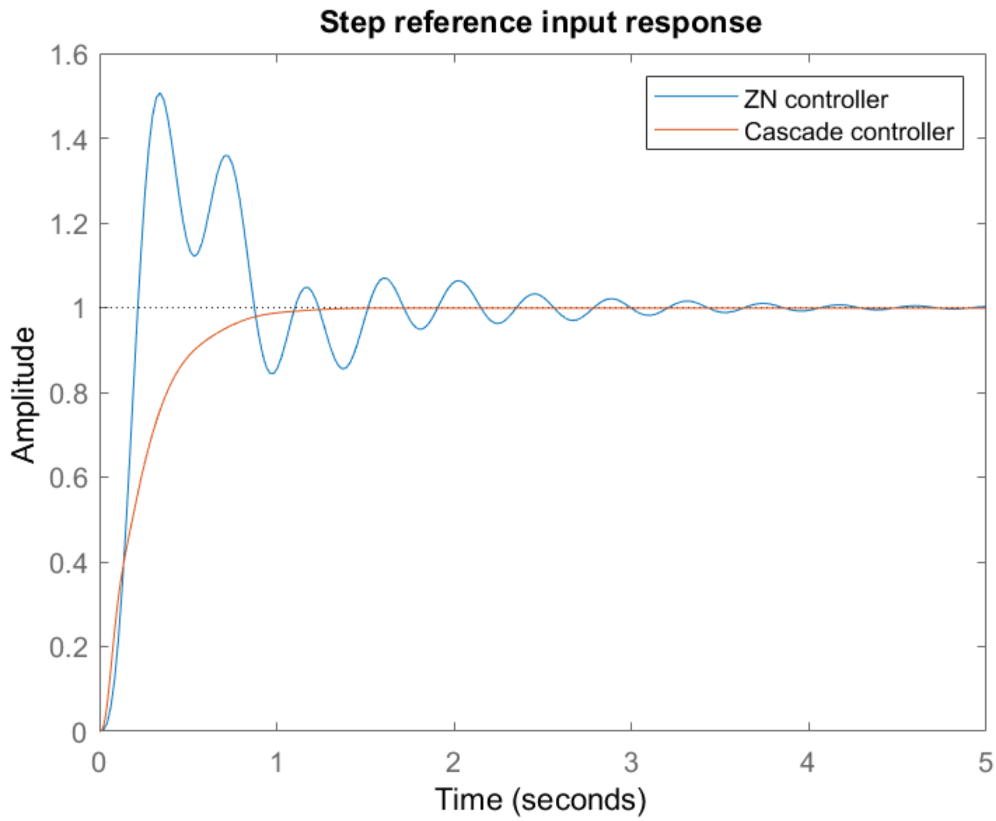
Thus we have

$$V_{cascade} = \frac{Y(s)}{W(s)} = \frac{G_1(s)}{s + G_1(s) D_c' \cdot s + D_c D_c' G_1(s)}$$

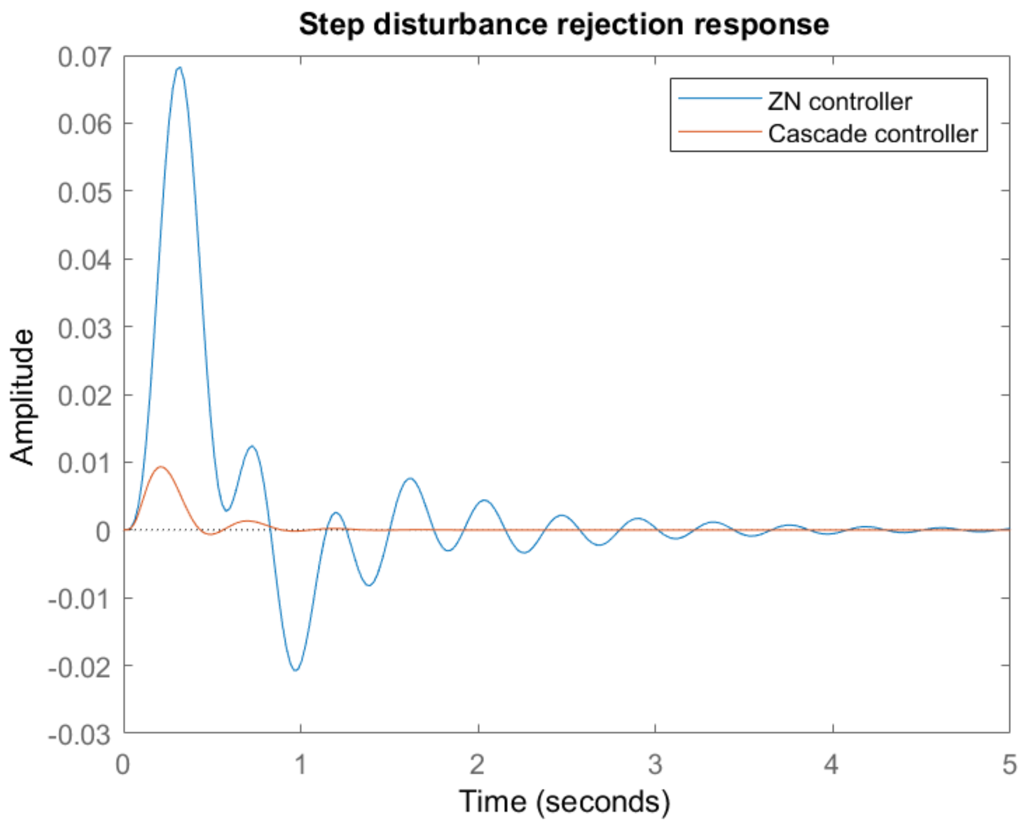
$$= \frac{3654.6 \cdot s^5 (s+43.33)^2 (s^2 + 6.82 \cdot s + 234.6)^2}{s^4 (s+43.33)^2 (s+4.446)(s^2 + 7.492 \cdot s + 185)(s^2 + 6.82 \cdot s + 234.6)^2 (s^2 + 38.21 \cdot s + 1980)}$$

2.3.6 Plot the closed-loop output for tracking a step reference signal for the cascade controller and the ZN controller in the same figure. Plot the closed-loop output for the rejection of a step disturbance for the cascade controller and the ZN controller in the same figure.

As shown below, the response of the system to a step reference input using the ZN and the Cascade controller:



As shown below, the rejection of the system to a step disturbance using the ZN and the Cascade controller:

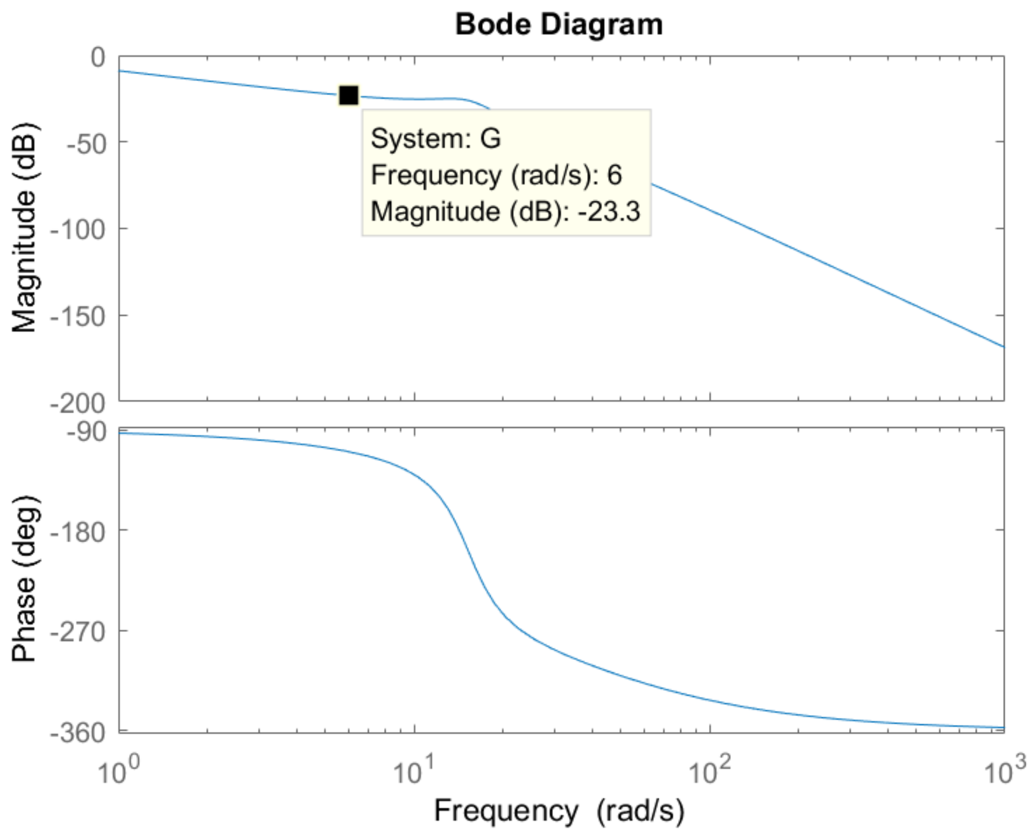


Module 3: Loop Shaping Method

3.1 Proportional controller

3.1.1 Give the value of kP and explain how you compute it from the Bode diagram of G. Give the values of gain margin and phase margin using the Bode diagram of the open-loop transfer function. Check your results using the command margin of Matlab.

We want to design a controller such that the crossover frequency is 6 rad/s. We start by drawing the Bode diagram:

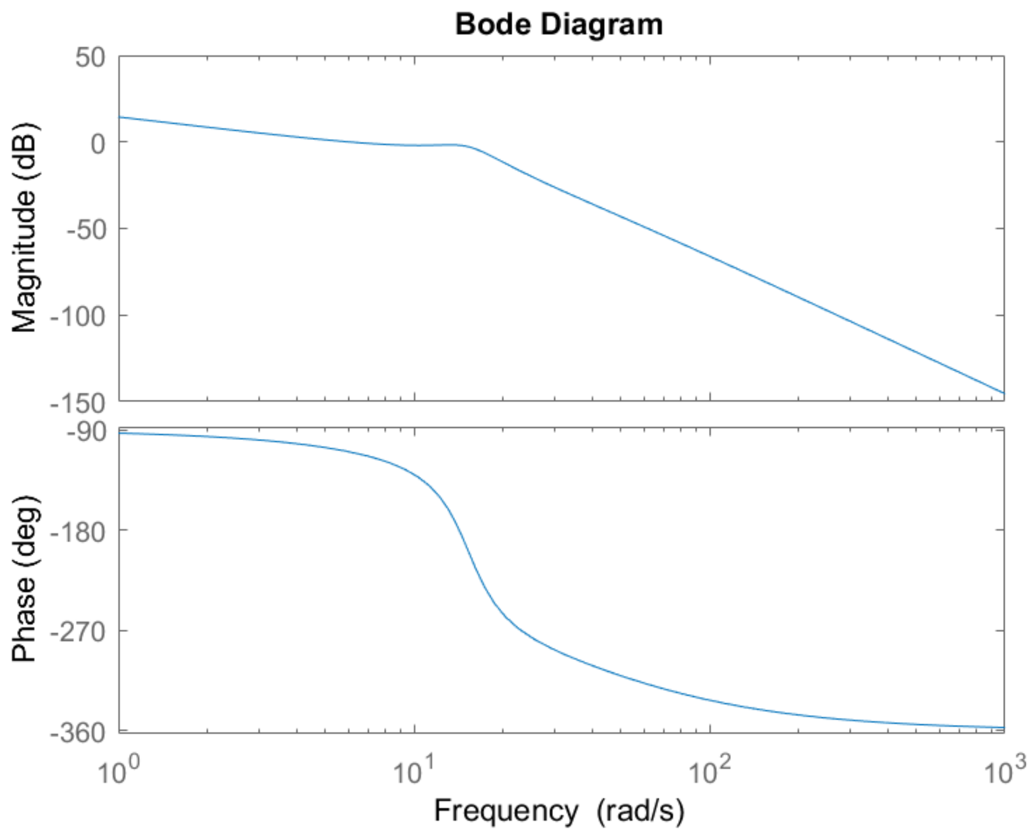


Using the *Data Cursor* tool, we can see that at 6 rad/s, the amplitude is 23.3 dB under the zero axis. As a result, we need to design k_p such that it lifts the plot 23.3 dB upper.

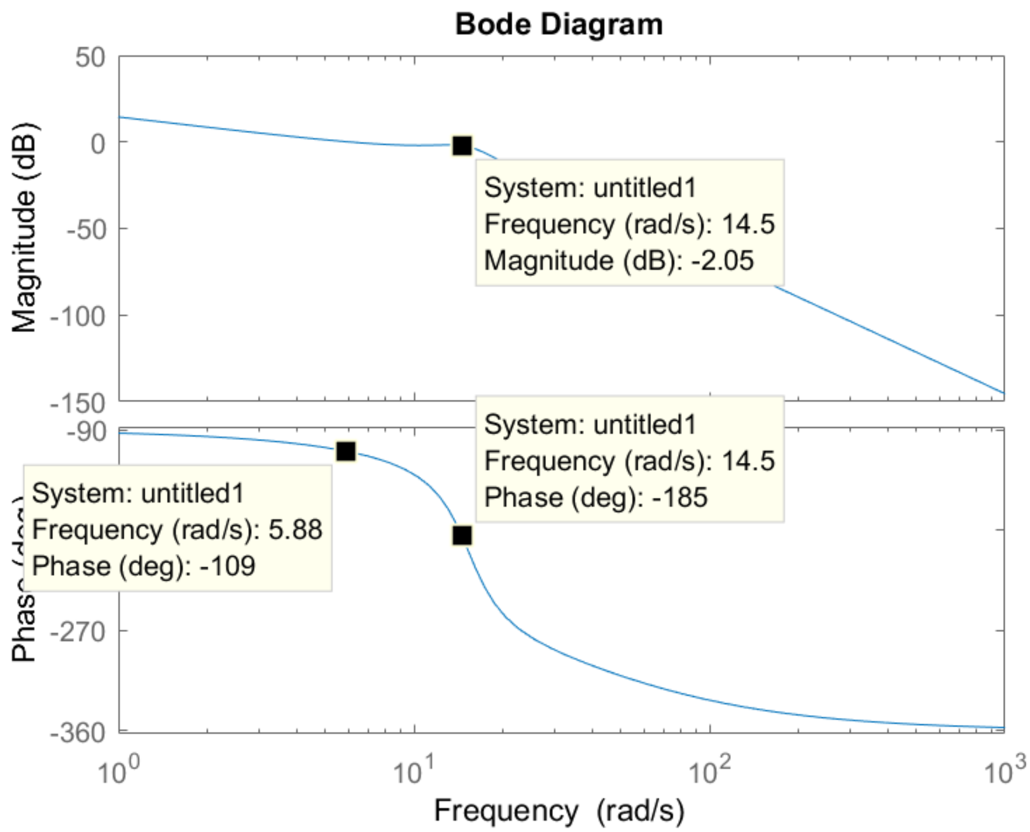
We need to solve te following equation:

$$20 \cdot \log_{10}(k_p) = 23.3 \Rightarrow k_p = 10^{\frac{23.3}{20}} \approx 14.62$$

We can now recompute the Bode diagram in order to control our results and measure the Gain and Phase margins:



The phase margin on the plot can be red by measuring the distance between the graph at the crossover frequency ($6\ rad/s$)and $-180°$. Using the same strategy, we can read the gain margin counting the difference between the actual position of the plot and $0dB$ at the frequency where the phase crosses the $-180°$ horizontal line (around $14.5\ rad/s$):



We can compare the Gain and Phase margins appearing on the plot by computing the matlab command:

$$[GM, PM] = margin(kp * G)$$

Thus, we obtain:

- $G_m = 1.2421\ dB$
- $P_m = 70.3418°$

3.2 Lead-Lag controller

3.2.1 Give the controller and explain in details how did you compute it.

We want to compute a controller which respect these specifications:

- Input step disturbance rejection
- Steady-state error of 0.0625 rad for ramp disturbance
- Set the crossover frequency at $\omega_c = 4\ rad/s$
- Minimum phase margin of 55°

As we are considering the disturbance, we shall establish a relation between $W(s)$ and $E(s)$:

$$\frac{E(s)}{W(s)} = -\frac{G(s)}{1 + G(s) \cdot D_c(s)}$$

A ramp disturbance can be translated as $W(s) = \frac{1}{s^2}$, therefore, using the *Final Value Theorem* the steady-state error can be computed as:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0} s\cdot W(s)\left\{-\frac{G(s)}{1+G(s)\cdot D_c(s)}\right\} = \lim_{s\to 0} s\frac{1}{s^2}\left\{-\frac{G(s)}{1+G(s)\cdot D_c(s)}\right\} = \frac{1}{K_v} = 0.0625\ rad$$

Thus, we find the *velocity constant* $K_v = \frac{1}{0.0625} = 16$,by rearranging the therms we obtain:

$$\lim_{s\to 0} -\frac{1}{s}\frac{1}{\frac{1}{G(s)}+D_c(s)} = \frac{1}{K_v} \Rightarrow K_v = \lim_{s\to 0} s(\frac{s}{G(s)} + D_c(s)) = \lim_{s\to 0} sD_c(s)$$

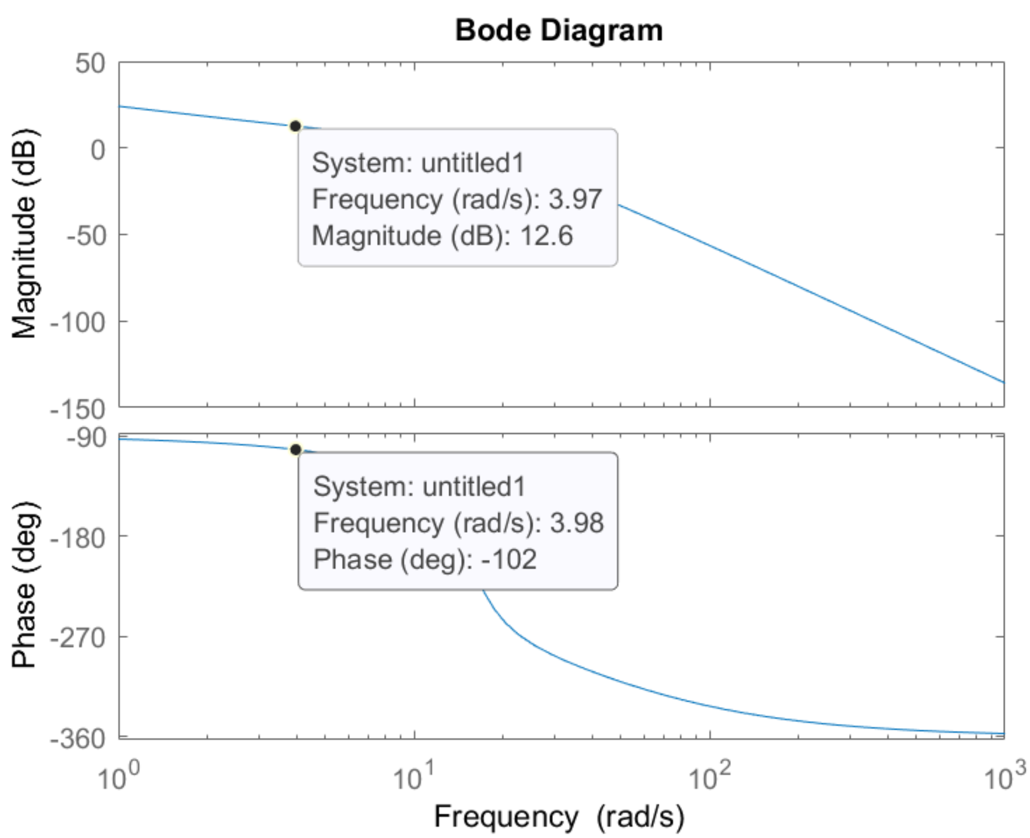
We want to design a *Lead-lag Controller*. As a result, the term $D_c(s)$ will have the following from:

$$D_c(s) = K\frac{1}{s^l}\prod_{i=1}^{m}\frac{1+\alpha_i\tau_i s}{1+\tau_i s}$$

Moreover, we want the *step input disturbance* to be rejected. As the $G(s)$ function already contain an *integrator* we done on't need to add in our $D_c(s)$ controller. This means that we can set $l = 0$.
We can now combine the two previous equation to calculate the Gain K:

$$K_v = \lim_{s\to 0} sD_c(s)G(s) = \lim_{s\to 0} sK\prod_{i=1}^{m}\frac{1+\alpha_i\tau_i s}{1+\tau_i s}G(s) = K\frac{3654.6}{43.33\cdot 234.6} \Rightarrow K = 44.5$$

We will now analyse the Bode diagram with our new calculated lead-lag controller: $D_c(s) = K = 44.5$. And then modify the values to obtain the correct crossover requency and phase margin.



As presented on the previous diagram, the graph need to be shifted around $-12.6\ dB$ upward to respect the desired qualifications. We can see that the phase margin is around 78°, which is already greater than 55°. As a consequence, we don't technically need to modify the margin. But for learning purposes, we will here lower the phase by 23°, in order to obtain a margin of exactly 55°.

Those two modifications can be done by setting the correct value(s) of α in the Lead-lag compensator. With the help of the equation given in the theory of the Lead-lag compensators in chapter 6, we have:

- Magnitude contribution, $\sqrt{c}$ (not given in decibel) at ω_c: $c = [\cdots] = \frac{1+(\alpha\tau\omega_c)^2}{1+(\tau\omega_c)^2}$
- Phase contribution, ϕ at ω_c: $\phi = [\cdots] = arg(1 + \alpha(\tau\omega_c)^2 + j(\alpha\tau\omega_c - \tau\omega_c))$
- We define p : $p = tan(\phi) = \frac{\alpha\tau\omega_c - \tau\omega_c}{1+\alpha(\tau\omega_c)^2}$
- We know that a *lead compensator* correspond to $\alpha > 1$ and a *lag compensator* correspond to $\alpha \in [0; 1]$

As our graph need to be lifted in phase, we want to introduce a lead in our system. As a consequence, we will choose $\alpha > 1$. We need to solve this equation:

$$(p^2 - c + 1)\alpha^2 + 2p^2c\alpha + p^2c^2 + c^2 - c = 0$$

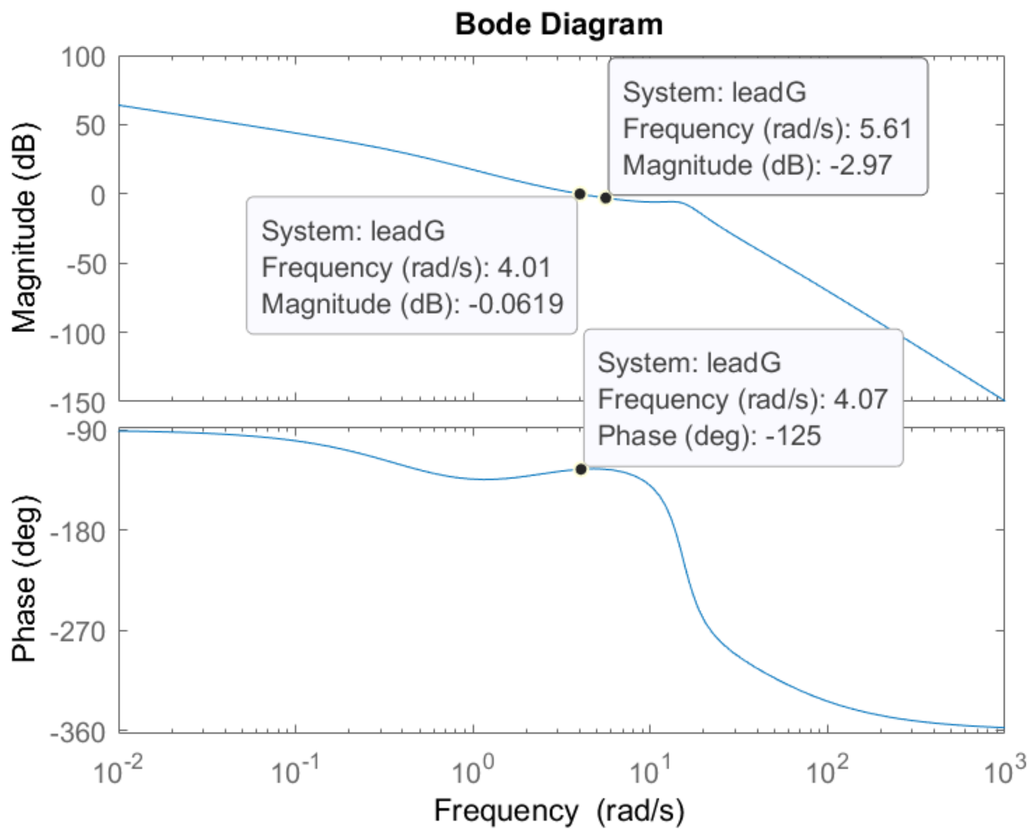
Rejecting the one solution that does not respect the $\alpha > 1$ condition, we find $\alpha = 0.205$ and $\tau = \frac{1}{\omega_c}\sqrt{\frac{1-c}{c-\alpha^2}} = 2.140$.

Finally our controller take the final form:

$$D_c(s) = K\frac{1+\alpha\tau s}{1+\tau s} = 44.5 \cdot \frac{1 + 2.140 \cdot 0.205 \cdot s}{1 + 2.140 \cdot s}$$

3.2.2 Check your results (closed-loop bandwidth, phase margin).

We can now plot the Bode diagram of $Dc(s)G(s)$ using our new lead compensator:



The *closed-loop bandwidth* is defined as the frequency where the graph reaches the $-3\ dB$ value. We can read the bandwidth directly on the graph: $BW = 5.61\ rad$.

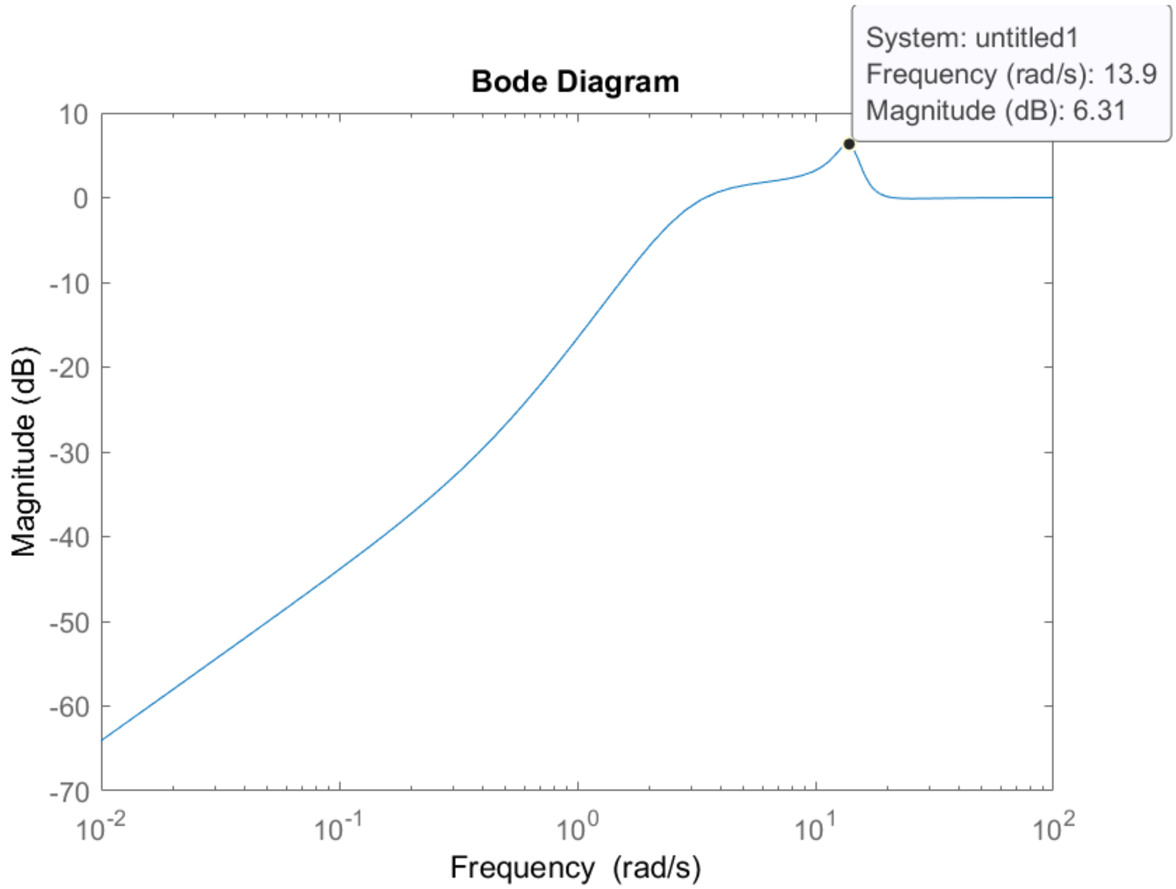
The given margin computed with *Matlab* using $[GM, PM] = margin(D_c * G)$ are:

- $GM = 1.9307$
- $PM = 54.591$

The plot and the Matlab calculations both confirm the exactitude of our developement.

3.2.3 Compute the modulus margin from the magnitude Bode diagram of the closed-loop sensitivity function.

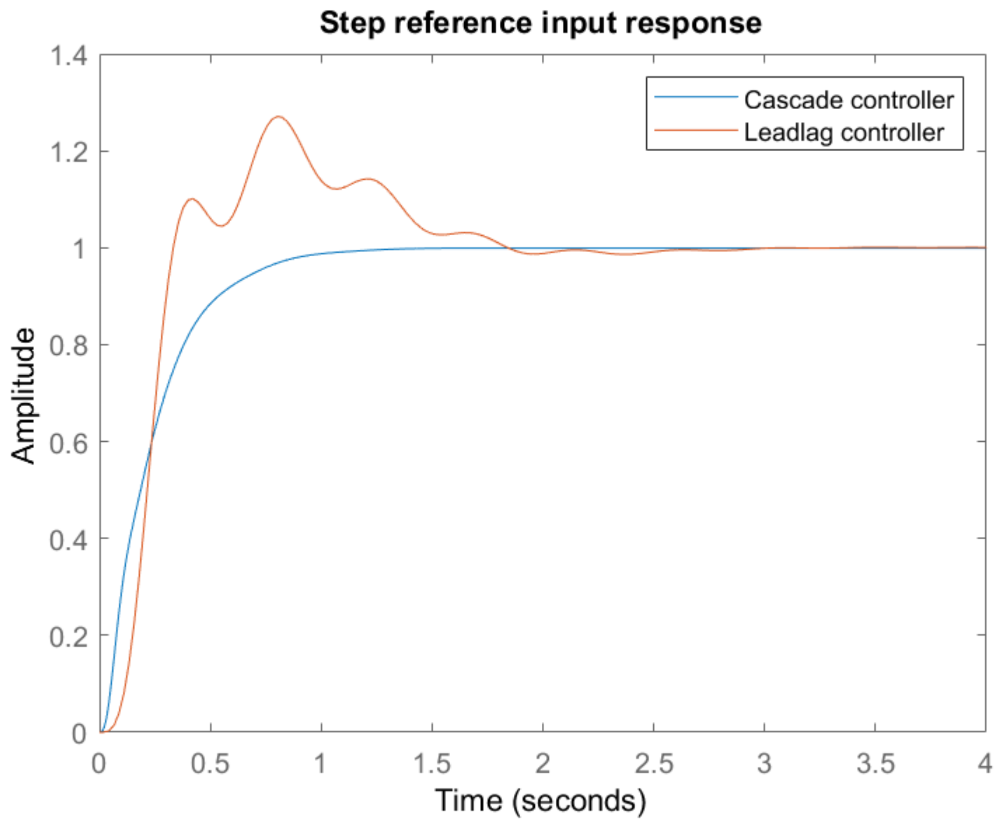
The modulus margin is related to the maximal value of the Bode plot of the closed loop sensitivity equation: $\frac{1}{1+D_c(s)G(s)}$



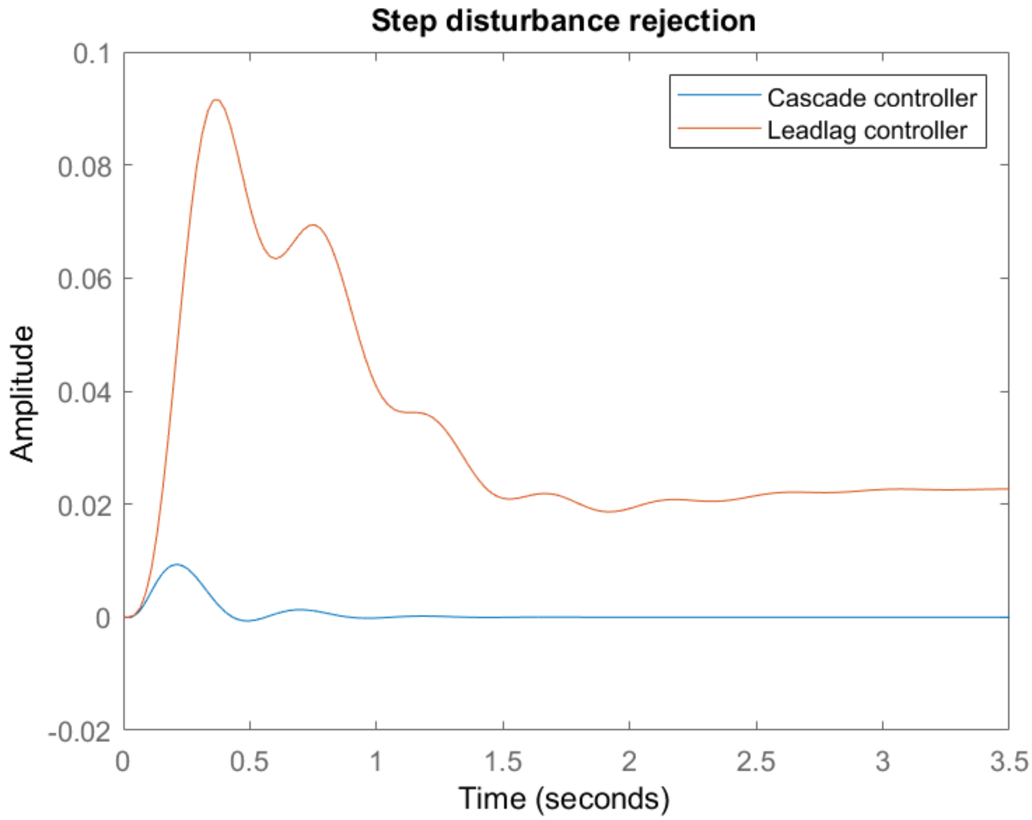
The *Modulus Margin* is computed by taking the inverse of the peak value red on the previous plot (note that the value should be converted from decibel to standard units): $MM = \frac{1}{10^{\frac{6.31}{20}}} = 0.484$

3.3 Comparison with cascade controller

3.3.1 Compare the tracking step response of the lead-lag controller and the cascade controller (plot both responses in the same figure).



3.3.2 Compare the disturbance step response of the lead-lag controller and the cascade controller (plot both responses in the same figure).



Module 4: State-Space Method

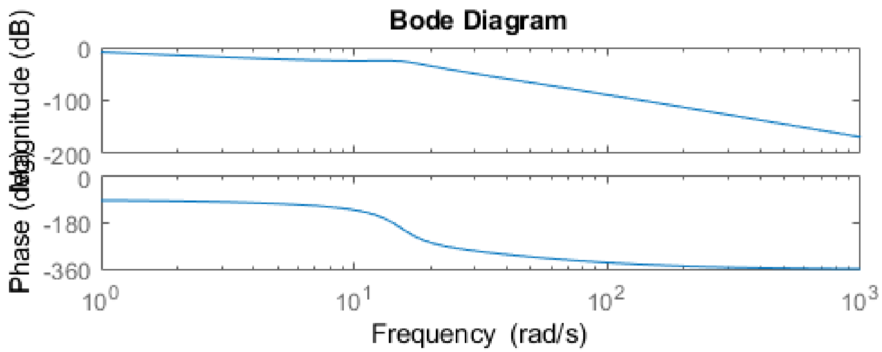
4.1 State-Space Model

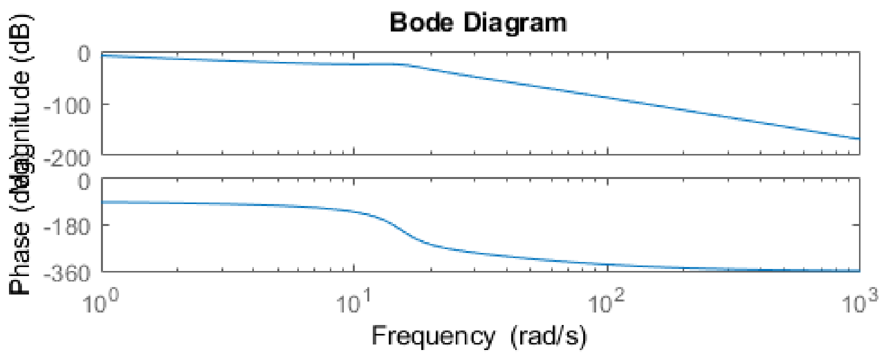
4.1.1 Give the state space equation of the system and the state space model in matrix form.

$$\mathbf{A} = \begin{bmatrix} \frac{-K_g^2 K_m^2}{R_m J_m} - \frac{b}{J_m} & 0 & 0 & \frac{K_s}{J_m} \\ 1 & 0 & 0 & 0 \\ \frac{K_g^2 K_m^2}{R_m J_m} + \frac{b}{J_m} & 0 & 0 & \frac{-K_s(J_m+J_{Br})}{J_m J_{Br}} \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \frac{K_G K_m K_a}{R_m J_m} \\ 0 \\ \frac{-K_G K_m K_a}{R_m J_m} \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} \qquad \mathbf{D} = 0$$

4.1.2 Validate your model by comparison of the Bode diagram of the state-space model and G(s) of the first module.





4.1.3 Is the system controllable? Why?

The matrix of controllability reads:

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$$

The determinant of this matrix is $\begin{vmatrix} B & AB & A^2B & A^3B \end{vmatrix} = 4,34 \cdot 10^9 \neq 0$. As a result, the system is controllable.

4.1.4 Is the system observable? Why?

The matrix of observability reads:

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$$

The determinant of this matrix is $\begin{vmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{vmatrix} = 4,1 \cdot 10^4 \neq 0$. As a result, the system is observable.

4.2 State-Space Controller Design

4.2.1 Give the codes for state-feedback controller design and the final numerical values of the controller K.

After computing the poles of $s^2 + 2\zeta\omega_n s + \omega_n^2$, we obtained

- $s_1 = -5.6 - 4.2i$
- $s_2 = -5.6 + 4.2i$

We chose the two fast poles shown below:

- -70
- -70

We compute **K** using the the following Matlab command:

$P = [-70; 70; -5.6 - 4.2i; -5.6 + 4.2i];$
$K = acker(A, B, P);$

We obtain

$$\mathbf{K} = \begin{bmatrix} 14.1124 & 65.6978 & 8.5074 & -266.3642 \end{bmatrix}$$

4.2.2 Give the codes for state estimator design and the final numerical values for L.

We can obtain the estimator using the Matlab commands:

$$L = acker(A', B', 5 * P)$$

L reads:

$$\mathbf{L} = \begin{bmatrix} 2.6114 \cdot 10^5 & 0.0546 \cdot 10^5 & -1.3414 \cdot 10^5 & -0.0476 \cdot 10^5 \end{bmatrix}$$

4.2.3 Give the code for computing the feedforward gain and its final numerical value.

We can obtain the estimator using the Matlab commands:

$Ccl = (C, 0, 0, 0, 0);$
$Acl = [A, -B * K; L' * C, A - B * K - L' * C];$
$Nb = -inv(Ccl * inv(Acl) * [B; B]);$

$\overline{\mathbf{N}}$ reads:

$$\overline{\mathbf{N}} = 0.0268$$

4.2.4 Give the state-space representation for the closed-loop system between r and y. Give the numerical values of the transfer function.

The transfer function in zpk format reads:

$$T(s) = C_{cl}(sI - A_{cl})^{-1}B_{cl} = \frac{98 \cdot (s + 350)^2 \cdot (s^2 + 56 \cdot s + 1225)}{(s + 350)^2(s + 43.36) \cdot (s + 0.009543) \cdot (s^2 + 6.776 \cdot s + 236.8) \cdot (s^2 + 56 \cdot s + 1225)}$$

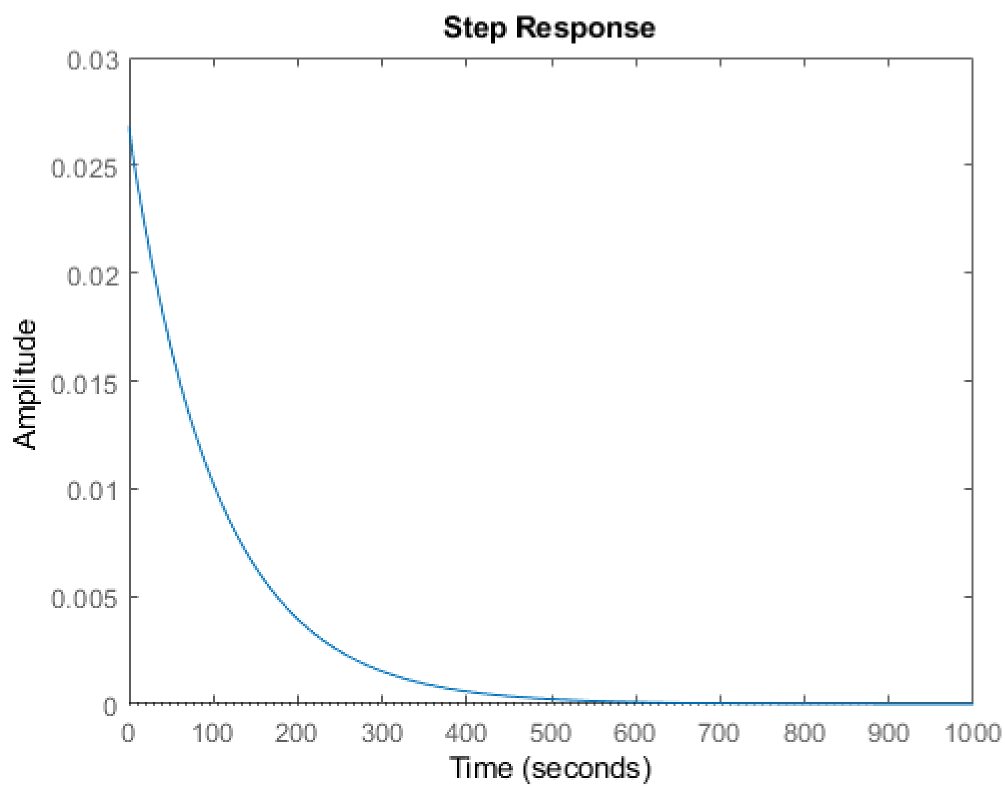
4.2.5 Give the state-space representation for the closed-loop system between r and u. Give the numerical values of the transfer function.

$$U(s) = \begin{bmatrix} 0 & 0 & 0 & 0 & -\mathbf{K} \end{bmatrix} (sI - A_{cl})^{-1} B_{cl} + \overline{N}$$

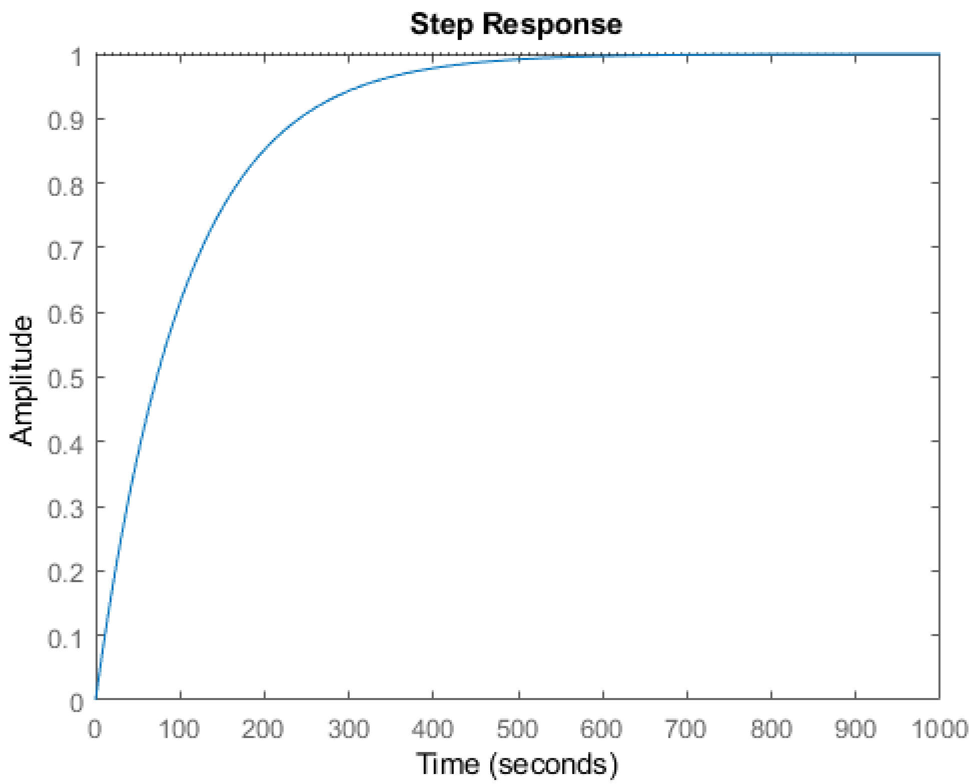
$$= \frac{0.026815 \cdot s(s+350)^2 \cdot (s+43.33) \cdot (s^2+6.82 \cdot s+234.6) \cdot (s^2+56 \cdot s+1225)}{(s+350)^2 \cdot (s+43.36) \cdot (s+0.009543) \cdot (s^2+6.776 \cdot s+236.8) \cdot (s^2+56 \cdot s+1225)}$$

4.2.6 Plot the control signal u(t) and the output y(t) for a unit step reference signal.

The control signal reads as shown:



This is the output for a step reference signal:



4.3 State-Space Controller with integrator

4.3.1 Give the codes and the numerical values for the augmented model.

We obtained the augmented model using the following Matlab sequence:

```
Z = [0;0;0;0];
Abarre = [A Z;-C 0];
Bbarre = [B; 0];
```

The result reads as sown below:

$$\mathbf{Z} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \overline{\mathbf{A}} = \begin{bmatrix} 50.1462 & 0 & 0 & 327.4108 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 50.1462 & 0 & 0-530.1135 & 0 & \\ 0 & 0 & 1.0000 & 0 & 0 \\ 0 & -1.0000 & 0 & -1.0000 & 0 \end{bmatrix} \qquad \overline{\mathbf{B}} = \begin{bmatrix} 18.0294 \\ 0 \\ -18.0294 \\ 0 \\ 0 \end{bmatrix}$$

4.3.2 Give the codes for state-feedback controller design and the final controller K.

The codes for calculating K reads:

```
K = acker(Abarre,Bbarre,p);
```

This gives the following value for K:

$\mathbf{K} = [0.8141\ 19.3933\ 0.4783\ -19.3149\ -45.2510]$

4.3.3 Give the codes for state estimator design and the final gain L.

We obtained the augmented model using the following Matlab code:

L = acker(A',C',pe)

The result reads as sown below:

$\mathbf{L} = [24.7121\ -9.3141\ -182.7497\ 4.2264\ 0.0632]$

4.3.4 Give the state-space representation for the closed-loop system between r and y. Give the numerical values of the transfer function.

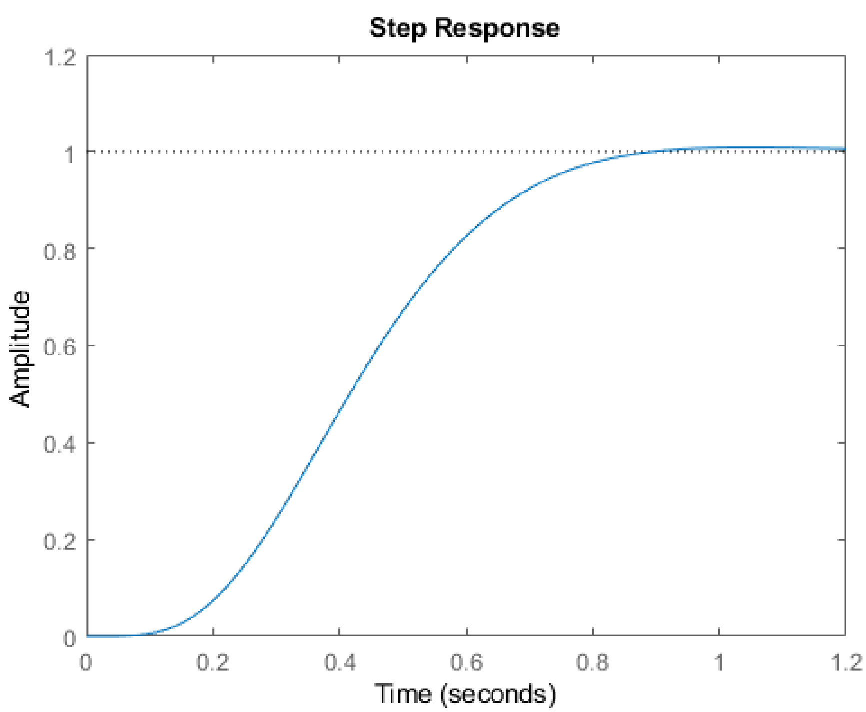
$$T(s) = \frac{1.6538 \cdot 10^5 (s+75)^3 \cdot (s^2 + 56 \cdot s + 1225)}{(s+75)^3 (s+15)^3 (s^2 + 11.2 \cdot s + 49)(s^2 + 56 \cdot s + 1225)}$$

4.3.5 Give the state-space representation for the closed-loop system between r and u. Give the numerical values of the transfer function.

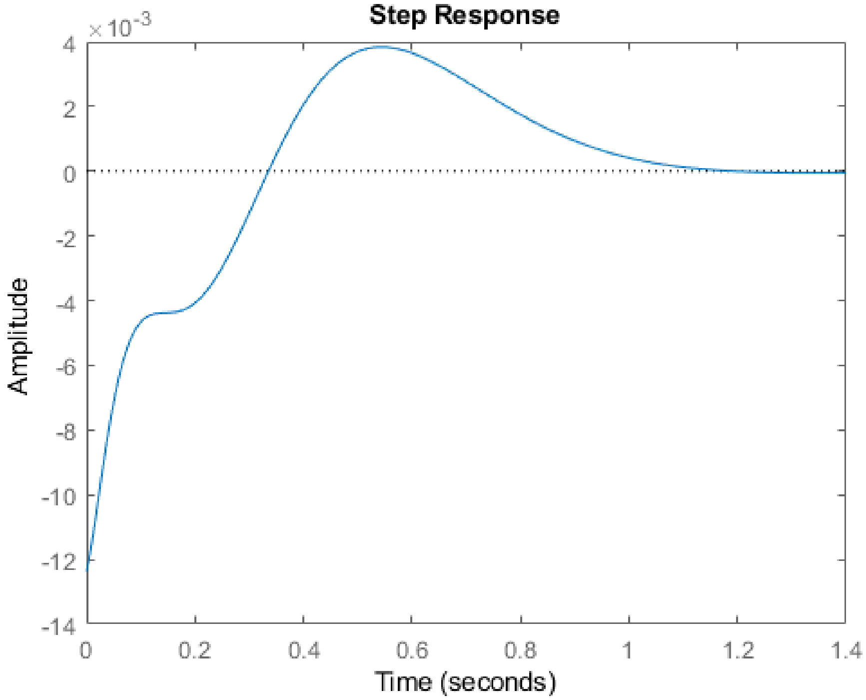
$$U(s) = \frac{-0.012382(s+75)^3 (s+43.33)(s+7.535 \cdot 10^-7)(s-7.535 \cdot 10^-7)(s^2+6.82 \cdot s+234.6)(s^2+56 \cdot s+1225)}{(s+75)^3 (s+15)^3 (s^2+11.2 \cdot s+49)(s^2+56 \cdot s+1225)}$$

4.3.6 Plot the control signal u(t) and the output y(t) for a unit step reference signal.

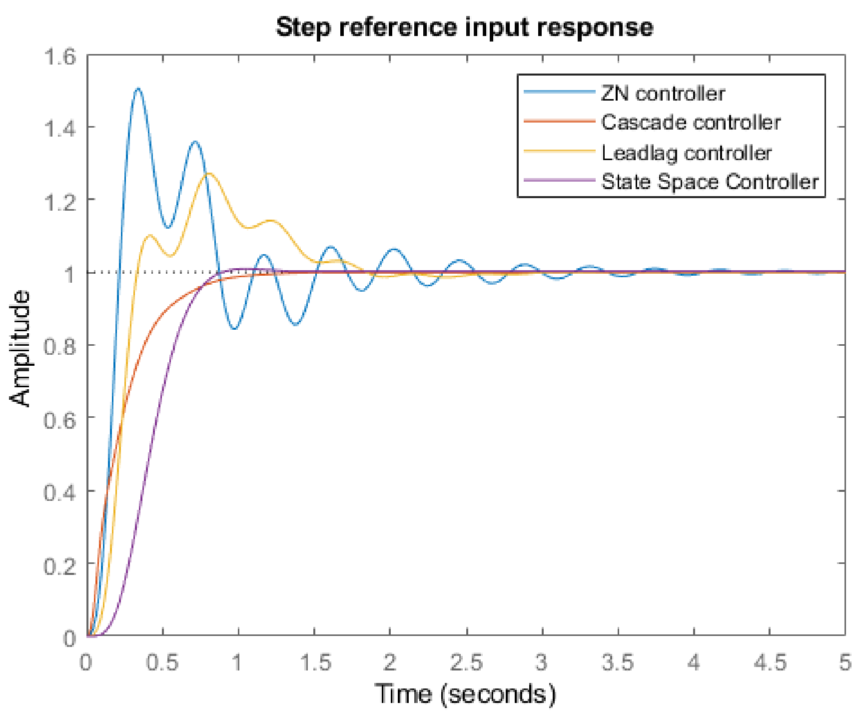
The control signal reads as shown:



This is the output for a step reference signal:



4.3.7 Compare the four methods: ZN method, cascade PID, loop shaping (with integrator) and state space (with integrator) in terms of performance in tracking by superposition of the time responses.



4.3.8 Compare these methods in terms of the facility and clarity of the design method, their advantages and disadvantages.

The Ziegler-Nicholson methods are easy to calculate and implement. Altough their implementation converges. The transient response induces a large overshoot and a lot of oscillation. The result of the lead lag are a bit better but they are a lot more complicated to compute.

The best result were achieved by the Cascade and the States Space controllers. Moreover the cascade controller is easier to calculate than the State Space.